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Summary of Research

(Nov. 1, 1966, - April 30, 1967)

The major goals of this research are to generate a deep understanding of communication channels and sources and to use this understanding in the development of reliable, efficient communication techniques.

1. Convolutional Codes and Their Decoding Techniques

Further results have been achieved in the error performance possible with systematic convolutional codes. It was shown earlier by Yudkin[1] that an error probability $P_e \leq \exp{-NE_{cv}(R)}$ could be achieved with non-systematic codes. In this expression N is the code constraint length in channel digits, R is the code rate, and $E_{cv}(R)$ is a function of the rate which is considerably larger than the corresponding exponent for block codes. The new results[2] show that the same expression is valid for a systematic code if N is taken as 1+Lm where L is the constraint length in information symbols and m is the number of check digits per information symbol (for the nonsystematic code, N would be (m+1)L). Work is progressing to find out if the same result is valid for sequential decoding and if an analogous result holds when different check digits have different constraint lengths. These questions are relevant in studies of how to ameliorate the effect of error propagation in sequential decoding.

The technique of semidefinite decoding, proposed by one of the investigators as a method of making partial use of previous decoding decisions in a way that avoids the possibility of infinite propagation of a decoding error, has been simulated for a binary symmetric channel by L. A. Frasco[3]. The simulation showed that for the classes of codes studied; namely the uniform codes, the self-orthogonal codes, and the Massey trial-and-error codes; the average decoding error probability was essentially a monotonically decreasing function of the number of semidefinite decoding stages. This probability approaches its minimum as the semidefinite decoder approaches the conventional feedback decoder. This simulation suggests the general optimality of feedback decoding for a wide class of codes and channels and the possible use of the concept of semidefinite decoding as an analytical tool to establish this optimality.

Research has continued into the general study of error propagation in decoding convolutional codes. It has been found that classes of convolutional codes exist with particularly nasty error propagation characteristics. In particular, there exist classes of codes which are nearly optimal from the point of view of error correction in the absence of past decoding errors that can be driven into a propagation condition by a bad "burst" of channel noise and will continue in the propagation condition until driven therefrom by another bad "burst" of channel noise. Study of these codes has led to the formalization of the concept of "pseudo-linearity" for convolutional decoders. This concept has proved useful in studying error propagation.

In connection with the investigation of error propagation, research was undertaken into the characteristics of <u>definite decoding</u>, i.e. of decoding convolutional codes without making any use of past decoding decisions. Success has been achieved in showing that there exist fixed codes of rate $R = \frac{1}{2}$ such that the ratio of definite decoding minimum distance to constraint length is at least .034. This result was established using Gilbert-bound type arguments by R. W. Kolor[4]. Codes with this distance-constraint-length-ratio property were known to exist previously only for the impractical time-varying convolutional codes and only for rates less than $R = \frac{1}{2}$.

W. Wilder has completed the task of proving a Plotkin-type upper bound for convolutional codes over an arbitrary finite field and with arbitrary code rates[5]. His result shows further that the same upper bound applies to the class of nonlinear tree codes known as "complementary codes". Several classes of codes that meet the bound with equality have also been derived.

2. Decoding BCH Codes

An investigation has been made into algebraic methods for decoding the Bose-Chaudhuri-Hocquenghem (BCH) codes. It was first realized that the major problem in decoding the BCH codes, namely the solution for the error locations from the weighted power sum symmetric functions, is formally equivalent to a problem of shift-register synthesis, i.e. finding the shortest linear feedback-shift-register (FSR) that can generate a specified sequence of digits. An FSR synthesis algorithm was developed which solves this problem in a simple and effective manner(6). When considered as a decoding procedure for the BCH codes, the FSR synthesis algorithm becomes essentially the same as the Berlekamp "iterative algorithm"(7), differing only by elimination of one minor test required in the latter. The FSR synthesis approach casts a new intuitive light into this latter procedure and also seems well adapted to certain other problems in automata minimization and redundancy reduction for data sources.

3. Optical Communciation Channels

The investigation of optical channels from the vantage point of information and detection theory has continued to focus upon the effects of atmospheric turbulence. Experimental studies of atmospheric propagation have continued(8). Also, a channel model applicable to "focused beams" has been developed beyond the point necessary for a communication analysis(9).

We have begun to shift our emphasis from the development of channel models to the study of these models. Our principal conclusions thus far have been that, subject to some reasonable constraints, the presence of atmospheric turbulence does not reduce the channel capacity that would exist in its absence and that, to be efficient, a receiver must exploit the spatial diversity that is contained within the receiver aperture.[10] An analytical development of reasonably simple receivers which exploit this diversity is now in progress.

Three doctoral level investigations have reached the proposal stage during the past six months. One of these is concerned with the application of estimation theory to the problem of high resolution astronomy (or surveillance) through the turbulent atmosphere. Preliminary results suggest that significant gains can be realized through the data processing techniques suggested by estimation theory[11]. The second investigation pertains to the fundamental limitations upon the transmission of information by combined temporal and spatial modulation, e.g. by a sequence of "images." Of particular concern is the interplay between time, bandwidth, aperture size, and background noise[12]. The remaining investigation is directed towards a fundamental examination of the role of quantum theory in communication theory[13]. The central issue in this investigation is the determination of the limitations imposed upon reliable communication by quantum effects and a determination of the receivers & waveforms which attain these limits.

4. Coding Theorems

A Ph.D. thesis on the topic of parallel channels with no crosstalk but with statistically dependent noise has been completed[14]. Major results concerned the adequacy of state models for such channels, the determination of capacity, and the establishment of a coding theorem. Particular attention was given to the asymptatic behavior of the error exponent in the limit of a large number of parallel channels all in the same state. The results are particularly applicable to frequency multiplexed systems using coding and subject to broadband fading.

Another Ph.D. thesis, on bandlimited fading dispersive channels, is nearing completion[15]. It has been found by numerical analysis that at rates close to the infinite bandwidth capacity, very large bandwidths are required to approach the infinite bandwidth exponent, but as the rate is decreased, the bandwidth required to approach the infinite bandwidth exponent is reduced to quite reasonable values. It has also been found that the error probability exponent is maximized by using a finite (and often very small) number of amplitude levels. This is, of course, useful in the instrumentation of coding techniques for such channels.

A third Ph.D. thesis nearing completion is on the topic of Coding Theorems for Unsynchronized Noisy Channels[16]. It has been found that at data rates close to capacity the lack of synchronization does not effect the error probability exponent. For sufficiently symmetric channels the exponent without synchronization is equal to the exponent with synchronization at all rates, although at low rates the set of code words used in one frame must depend on the set used in the previous frame. Some unsynchronized minimum distance bounds are established for binary codes which generalize earlier work on comma free codes.

Finally a general coding theorem on channels with memory has been developed.[17]

5. Source Coding with a Distortion Measure

A number of results habe been established concerning the relationship of fixed length to variable length source codes subject to a distortion measure.[18] The distinction is important where the distortion measure takes on infinite values. Physically, an infinite distortion is used in cases where one wants to limit both average distortion and peak distortion. Other results concern source coding techniques and the behavior of quantizers in source coding.

6. Codes and Automata

Some study has been made of connections between results in coding theory and those of automata theory Cyclic codes and convolutional codes respectively have been related to the zero-input and zero-state response respectively of linear automata. The notions of state-observability and state-controllability in automata has been related to burst-correcting convolutional codes[19].

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